

# A new picture on (3+1)D topological mass mechanism

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## Abstract

We present a class of mappings between the fields of the Cremmer-Sherk and pure BF models in 4D. These mappings are established by two distinct procedures. First a mapping of their actions is produced iteratively resulting in an expansion of the fields of one model in terms of progressively higher derivatives of the other model fields. Secondly an exact mapping is introduced by mapping their quantum correlation functions. The equivalence of both procedures is shown by resorting to the invariance under field scale transformations of the topological action. Related equivalences in 5D and 3D are discussed. A cohomological argument is presented to provide consistency of the iterative mapping.

## 1 Introduction

The search for ultraviolet renormalizable models has always been one of the most attractive and relevant aspects of quantum field theory. As it is well known, the program of describing the eletro-weak interactions in the language of QFT is based on the construction of the Higgs mechanism for mass generation of the vector bosons. However, this mechanism relies on the existence of a scalar particle, the Higgs boson, whose experimental evidence is still lacking.

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In this context, the topological mechanism for mass generation is attractive, since it provides masses for the gauge vector bosons without the explicit introduction of new scalar fields. For example, in three-dimensional spacetime, the topological non-Abelian Chern-Simons term generates mass for the Yang-Mills fields while preserving the exact gauge invariance [1]. In four dimensions the topological mass generation mechanism occurs in the case of an anti-symmetric tensorial field  $B_{\mu\nu}$ . It has been shown that the Cremmer-Sherk action gives a massive pole to the vector gauge field in the Abelian context. This model is described by the action [2]

$$S = \int d^4x \left( -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{m}{4} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} B_{\rho\sigma} \right). \quad (1.1)$$

Indeed, as it was shown [3], this model exists only in the Abelian version. In fact, possible non-Abelian generalizations of the action (1.1), will necessarily require non-renormalizable couplings, as in [4], or the introduction of extra fields [5]. Anti-symmetric fields in four dimensions also deserve attention since they appear naturally by integrating out the fermionic degrees of freedom in favor of bosonic fields in bosonization approaches. The fermionic current turns out to be expressed in terms of derivatives of the tensorial field as a topologically conserved current. The coupling of this current to the gauge field leads to terms in the effective action similar to the last one in (1.1).

An important property of the three dimensional Yang-Mills type actions, in the presence of the Chern-Simons term, was pointed out in [6], i.e., it can be cast in the form of a pure Chern-Simons action through a nonlinear covariant redefinition of the gauge connection. The quantum consequences of this fact were investigated in the BRST framework yielding an algebraic proof of the finiteness of the Yang-Mills action with topological mass.

In this work we present a recursive mapping between Cremmer-Sherk's action and the pure topological  $BF$  model. With this the fields of one action are expressed as a series of progressively higher derivatives of the other model fields. This mapping is also established along a different line in which the propagators of one action are reproduced using a closed expression in terms of the other action fields. This exposes the non-local nature of the mapping. Related mappings in higher and lower dimensions are discussed. A cohomological argument is presented to give consistency to the recursive mapping.

## 2 Mapping the fields

The aim of this section is to establish the classical equivalence between the Cremmer-Sherk's action and the pure  $BF$  theory, *i. e.*, that the first action can be mapped to the second one through a redefinition of the gauge field. Following the same steps of the three-dimensional case [6], we search for a redefinition of the fields  $A_\mu$  and  $B_{\mu\nu}$  as a series in powers of  $1/m$  in terms of the fields  $\hat{A}_\mu$  and  $\hat{B}_{\mu\nu}$  in such a way that the relation below is valid<sup>1</sup>:

$$\mathcal{S}_M(A) + \mathcal{S}_H(B) + \mathcal{S}_{BF}(A, B) = \mathcal{S}_{BF}(\hat{A}, \hat{B}), \quad (2.2)$$

where

$$\mathcal{S}_M(A) = -\frac{1}{4} \int d^4x (F^{\mu\nu} F_{\mu\nu}), \quad (2.3)$$

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<sup>1</sup>We work in the Minkovski spacetime so that  $\varepsilon^{\mu\nu\rho\sigma} \varepsilon_{\mu\nu\alpha\beta} = -2 (\delta_\alpha^\rho \delta_\beta^\sigma - \delta_\beta^\rho \delta_\alpha^\sigma)$ . We use  $\varepsilon^{0123} = 1$  and  $\text{diag } \eta_{\mu\nu} = (1, -1, -1, -1)$ .

$$\mathcal{S}_H(B) = +\frac{1}{12} \int d^4x (H_{\mu\nu\rho} H^{\mu\nu\rho}) , \quad (2.4)$$

$$\mathcal{S}_{BF}(A, B) = \frac{m}{4} \int d^4x (\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} B_{\rho\sigma}) \quad (2.5)$$

and the curvatures  $F_{\mu\nu}$  and  $H_{\mu\nu\rho}$  are the same given in (1.1), *i. e.*,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu} .$$

Indeed taking the field redefinitions in the form

$$\begin{aligned} \hat{A}_\mu &= A_\mu + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} \vartheta_\mu^n , \\ \hat{B}_{\mu\nu} &= B_{\mu\nu} + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} \phi_{\mu\nu}^n , \end{aligned} \quad (2.6)$$

the equality expressed in (2.2) is implemented by recursively fixing the terms. We find the expressions

$$\begin{aligned} \phi_{\mu\nu}^{2n+1} &= -\frac{b_{2n+1}}{2} \epsilon_{\mu\nu\alpha\beta} \square^n F^{\alpha\beta} , \\ \vartheta_\mu^{2n+1} &= \frac{c_{2n+1}}{3} \epsilon_{\mu\nu\alpha\beta} \square^n H^{\nu\alpha\beta} , \\ \phi_{\mu\nu}^{2n} &= b_{2n} \square^{n-1} \partial^\alpha H_{\alpha\mu\nu} , \\ \vartheta_\mu^{2n} &= c_{2n} \square^{n-1} \partial^\nu F_{\nu\mu} , \end{aligned} \quad (2.7)$$

where the constants are defined as

$$b_{2n+1} = -\sum_{j=1}^n c_{2j} b_{2(n-j)+1} , \quad (2.8)$$

$$c_{2n+1} = -\sum_{j=1}^n b_{2j} c_{2(n-j)+1} , \quad (2.9)$$

$$b_{2n} = B_n \left[ 2 \sum_{j=1}^n b_{2j-1} c_{2(n-j)+1} - \sum_{j=1}^{n-1} b_{2j} c_{2(n-j)} \right] , \quad (2.10)$$

$$c_{2n} = (1 - B_n) \left[ 2 \sum_{j=1}^n b_{2j-1} c_{2(n-j)+1} + \sum_{j=1}^{n-1} b_{2j} c_{2(n-j)} \right] . \quad (2.11)$$

Here  $B_n$  are arbitrary constants introduced at each even step of the process while  $b_1 = -1$  and  $c_1 = -1/2$ .

The first terms can be expressed as

$$\begin{aligned}
\vartheta_\mu^1 &= \frac{-1}{6} \varepsilon^{\mu\nu\alpha\beta} H_{\nu\alpha\beta}, \\
\vartheta_\mu^2 &= (1 - B_1) \partial_\nu F^{\nu\mu}, \\
\vartheta_\mu^3 &= \frac{B_1}{6} \varepsilon_{\mu\nu\alpha\beta} \square H^{\nu\alpha\beta}, \\
\vartheta_\mu^4 &= (-1 - B_1 + B_1^2)(1 - B_2) \square \partial_\nu F^{\nu\mu}, \\
\vartheta_\mu^5 &= \frac{-B_1^2 - B_2 - B_2 B_1 + B_2 B_1^2}{6} \varepsilon_{\mu\nu\alpha\beta} \square^2 H^{\nu\alpha\beta}, \\
\vartheta_\mu^6 &= \left[ B_1^2 + B_2 - B_2 B_1 - 3B_2 B_1^2 + B_1 + 2 - B_1^3 + 2B_1^3 B_2 \right] (1 - B_3) \square^2 \partial_\nu F^{\nu\mu}, \\
\phi_{\mu\nu}^1 &= -\frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}, \\
\phi_{\mu\nu}^2 &= \frac{B_1}{3} \partial_\alpha H^{\alpha\mu\nu}, \\
\phi_{\mu\nu}^3 &= -\frac{1 - B_1}{2} \varepsilon_{\mu\nu\alpha\beta} \square F^{\alpha\beta}, \\
\phi_{\mu\nu}^4 &= \frac{B_2(-1 - B_1 + B_1^2)}{3} \square \partial_\alpha H^{\alpha\mu\nu}, \\
\phi_{\mu\nu}^5 &= \frac{2 - B_1 - B_2 - B_2 B_1 + B_2 B_1^2}{2} \varepsilon_{\mu\nu\alpha\beta} \square^2 F^{\alpha\beta}, \\
\phi_{\mu\nu}^6 &= \frac{(B_1^2 + B_2 - B_2 B_1 - 3B_2 B_1^2 + B_1 + 2 - B_1^3 + 2B_1^3 B_2) B_3}{3} \square^2 \partial_\alpha H^{\alpha\mu\nu}. \tag{2.12}
\end{aligned}$$

As we can see, up to the sixth order in the mass parameter, the coefficients,  $\phi_{\mu\nu}^n$  and  $\vartheta_\mu^n$ , shown in (2.12), depend on the three arbitrary dimensionless parameters,  $B_1$ ,  $B_2$  and  $B_3$ . In fact at each new even order in  $1/m$  a new arbitrary parameter is allowed to be introduced. As we shall see this is to be expected.

The formal series (2.6), which redefine the fields  $A_\mu$  and  $B_{\mu\nu}$ , give the mapping we were looking for.

Note that the gauge symmetry of the Cremmer-Sherk action is expressed as

$$\delta^g A_\mu = \partial_\mu \varepsilon, \quad \delta^g B_{\mu\nu} = 0 \tag{2.13}$$

and

$$\delta^t A_\mu = 0, \quad \delta^t B_{\mu\nu} = \partial_\mu \varepsilon_\nu - \partial_\nu \varepsilon_\mu, \tag{2.14}$$

while the BF topological action is invariant under analogous transformations

$$\delta^g \hat{A}_\mu = \partial_\mu \hat{\varepsilon}, \quad \delta^g \hat{B}_{\mu\nu} = 0 \tag{2.15}$$

and

$$\delta^t \hat{A}_\mu = 0, \quad \delta^t \hat{B}_{\mu\nu} = \partial_\mu \hat{\varepsilon}_\nu - \partial_\nu \hat{\varepsilon}_\mu. \tag{2.16}$$

The mapping (2.6,2.7) translates the gauge transformations of one pair of fields straightforwardly into the ones of the other pair in such a way that  $\hat{\varepsilon}$  and  $\hat{\varepsilon}_\mu$  are identified as  $\varepsilon$  and  $\varepsilon_\mu$ . This occurs since the higher order terms in (2.7) are gauge invariant. Indeed this result is natural but not

mandatory in this procedure. The mappings in (2.6) can be generalized by allowing for gauge dependence in the higher order terms. Due to the gauge symmetry such arbitrary terms would not contribute to the action and would not spoil the mapping.

## 2.1 Exact mapping

Let us express here the Cremmer-Sherk fields in terms of the pure BF fields using a new procedure. The Cremmer-Sherk propagators are given by

$$\begin{aligned}
\langle \imath \mathcal{T} B_{\mu\nu} B_{\alpha\beta} \rangle &= (P_{\mu\nu,\alpha\beta} + G_1 K_{\mu\nu,\alpha\beta}) \frac{2}{\square(\square + m^2)}, \\
\langle \imath \mathcal{T} B_{\mu\nu} A_\alpha \rangle &= - \langle \imath \mathcal{T} A_\alpha B_{\mu\nu} \rangle = (S_{\mu\nu\alpha}) \frac{m}{\square(\square + m^2)}, \\
\langle \imath \mathcal{T} A_\mu A_\alpha \rangle &= (P_{\mu,\nu} + G_2 K_{\mu,\nu}) \frac{-1}{\square(\square + m^2)}.
\end{aligned} \tag{2.17}$$

The pure BF propagators are given by

$$\begin{aligned}
\langle \imath \mathcal{T} \hat{B}_{\mu\nu} \hat{B}_{\alpha\beta} \rangle &= (\hat{G}_1 K_{\mu\nu,\alpha\beta}) \frac{2}{m\square}, \\
\langle \imath \mathcal{T} \hat{B}_{\mu\nu} \hat{A}_\alpha \rangle &= - \langle \imath \mathcal{T} \hat{A}_\alpha \hat{B}_{\mu\nu} \rangle = (S_{\mu\nu\alpha}) \frac{1}{m\square}, \\
\langle \imath \mathcal{T} \hat{A}_\mu \hat{A}_\alpha \rangle &= (\hat{G}_2 K_{\mu,\nu}) \frac{1}{m\square}.
\end{aligned} \tag{2.18}$$

Here the projectors are given by:

$$\begin{aligned}
P_{\mu\nu,\alpha\beta} &= \frac{1}{2} \delta_{\mu\nu,\alpha\beta} \square - \frac{1}{2} K_{\mu\nu,\alpha\beta}, \\
K_{\mu\nu,\alpha\beta} &= \delta_{\mu[\alpha} \partial_\nu \partial_{\beta]} - \delta_{\nu[\alpha} \partial_\mu \partial_{\beta]}, \\
S_{\mu\nu\alpha} &= \varepsilon_{\mu\nu\alpha\beta} \partial^\beta, \\
P_{\mu\nu} &= \delta_{\mu\nu} \square - \partial_\mu \partial_\nu, \\
K_{\mu\nu} &= \partial_\mu \partial_\nu.
\end{aligned} \tag{2.19}$$

The parameters  $G$  and  $\hat{G}$  are introduced to fix the gauge.

Let us try to express the fields as

$$\begin{aligned}
A_\mu &= (C_{AA} P_{\mu,\nu} + \delta_{\mu\nu}) \hat{A}^\nu + C_{AB} S_{\mu\alpha\beta} \hat{B}^{\alpha\beta}, \\
B_{\mu\nu} &= C_{BA} S_{\mu\nu\alpha} \hat{A}^\alpha + \left( C_{BB} P_{\mu\nu,\alpha\beta} + \frac{1}{2} \delta_{\mu\nu,\alpha\beta} \right) \hat{B}^{\alpha\beta}.
\end{aligned} \tag{2.20}$$

Computing the correlators of the Cremmer-Sherk field using this mapping and comparing with (2.17) the structure functions are fixed. They result to be given by the non-local operators

$$\begin{aligned}
C_{AA} &= \frac{2^{\frac{1}{2}} m^{\frac{1}{2}}}{\sigma \square} \left[ \frac{m - \sqrt{m^2 + \square}}{m^2 + \square} \right]^{\frac{1}{2}} - \frac{1}{\square}, \\
C_{BB} &= \frac{\sigma m^{\frac{1}{2}}}{2^{\frac{3}{2}} \square} \left[ \frac{m - \sqrt{m^2 + \square}}{m^2 + \square} \right]^{\frac{1}{2}} - \frac{1}{\square}, \\
C_{AB} &= \frac{\sigma m^{\frac{1}{2}}}{2^{\frac{5}{2}}} \left[ (m - \sqrt{m^2 + \square}) (m^2 + \square) \right]^{-\frac{1}{2}}, \\
C_{BA} &= \frac{2^{\frac{1}{2}} m^{\frac{1}{2}}}{\sigma} \left[ (m - \sqrt{m^2 + \square}) (m^2 + \square) \right]^{-\frac{1}{2}}.
\end{aligned} \tag{2.21}$$

Note that the non-local operators indeed map local fields of local models, the Cremmer-Sherk and pure BF models. Observe the presence of the arbitrary operator  $\sigma$  in these equations. Its presence should be expected since the set of transformations

$$\begin{aligned}
\hat{A} &\longrightarrow \sigma \hat{A}, \\
\hat{B} &\longrightarrow \frac{1}{\sigma} \hat{B},
\end{aligned} \tag{2.22}$$

does not affect any correlator of the BF model. The presence of  $\sigma$  in the mapping is due to the freedom in redefining the BF fields. This is the ultimate reason for the presence of the free parameters  $(B_1, B_2 \dots)$  in the mapping seen previously. In fact the exact inverse mapping is given by

$$\begin{aligned}
\hat{A}_\mu &= (\hat{C}_{AA} P_{\mu,\nu} + \delta_{\mu\nu}) A^\nu + \hat{C}_{AB} S_{\mu\alpha\beta} B^{\alpha\beta}, \\
\hat{B}_{\mu\nu} &= \hat{C}_{BA} S_{\mu\nu\alpha} A^\alpha + \left( \hat{C}_{BB} P_{\mu\nu,\alpha\beta} + \frac{1}{2} \delta_{\mu\nu, \alpha\beta} \right) B^{\alpha\beta},
\end{aligned} \tag{2.23}$$

with

$$\begin{aligned}
\hat{C}_{AA} &= \frac{\sigma}{2^{\frac{3}{2}} m^{\frac{1}{2}} \square} \left[ \frac{(m - \sqrt{m^2 + \square})^{\frac{3}{2}}}{(m^2 + \square)^{\frac{1}{2}}} \right] - \frac{1}{\square}, \\
\hat{C}_{BB} &= \frac{2^{\frac{1}{2}}}{\sigma m^{\frac{1}{2}} \square} \left[ \frac{(m - \sqrt{m^2 + \square})^{\frac{3}{2}}}{(m^2 + \square)^{\frac{1}{2}}} \right] - \frac{1}{\square}, \\
\hat{C}_{AB} &= \frac{\sigma}{2^{\frac{5}{2}} m^{\frac{1}{2}}} \left[ \frac{(m - \sqrt{m^2 + \square})}{(m^2 + \square)} \right]^{\frac{1}{2}}, \\
\hat{C}_{BA} &= \frac{2^{\frac{1}{2}}}{\sigma m^{\frac{1}{2}}} \left[ \frac{(m - \sqrt{m^2 + \square})}{(m^2 + \square)} \right]^{\frac{1}{2}}.
\end{aligned} \tag{2.24}$$

The iterative mapping may be retrieved by expanding the structure functions in terms of  $\frac{\square}{m^2}$  and, at the same time, expressing the operator  $\sigma$  in terms of arbitrary parameters as

$$\sigma = \sum_{n=0}^{\infty} C_n \left( \frac{\square}{m^2} \right)^n \tag{2.25}$$

and similarly to its inverse. With this equation (2.23) will reproduce equations (2.12). The independent parameters  $B_j$  are thus seen to owe their origin to the freedom in defining the operator  $\sigma$ . This allows for an independent parameter,  $C_j$ , to be introduced at each order in  $\square^j$ .

### 3 Dimensional reduction considerations

Let us consider in 5-D the model with the antisymmetric field which represents a direct generalization of the Maxwell-Chern-Simons model

$$S = \int d^5x \left( \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{m}{12} \varepsilon^{\mu\nu\alpha\beta\rho} B_{\mu\nu} H_{\alpha\beta\rho} \right). \quad (3.26)$$

The mapping of the field to the pure Chern-Simons field

$$\hat{S} = \int d^5x \left( \frac{m}{4} \varepsilon^{\mu\nu\alpha\beta\rho} B_{\mu\nu} H_{\alpha\beta\rho} \right) \quad (3.27)$$

is implemented by the transformation

$$\begin{aligned} B_{\mu\nu} = & \hat{B}_{\mu\nu} + \left[ \frac{2^{\frac{1}{2}}}{2m^{\frac{1}{2}}\square} \left[ \frac{(m - \sqrt{m^2 + \square})^{\frac{3}{2}}}{(m^2 + \square)^{\frac{1}{2}}} \right] - \frac{1}{\square} \right] P_{\mu\nu,\alpha\beta} \hat{B}^{\alpha\beta} \\ & - \frac{2^{\frac{1}{2}}}{12m^{\frac{1}{2}}} \left[ \frac{(m - \sqrt{m^2 + \square})}{(m^2 + \square)} \right]^{\frac{1}{2}} \varepsilon_{\mu\nu\alpha\beta\rho} \hat{H}^{\alpha\beta\rho}. \end{aligned} \quad (3.28)$$

This result is obtained repeating the argument of section 2-b in the 5-dimensional spacetime. Note that in this case the topological model does not present any freedom in rescaling the fields as occurs in 4D. The dimensional reduction of the model (3.26) by precluding any dependence on the variable  $x^4$  so that  $B_{\mu 4} := A_\mu$  leads to the Cremmer-Sherk model (1.1). Under similar considerations the topological model (3.27) is led to the pure BF model (2.5). Within this setting the mapping of the 5D fields (3.28) is reduced to (2.20) and (2.21) if we eliminate the freedom in the mapping by identifying  $\sigma = 2$ . Thus the dimensional reduction turns out to give a criterion to fix in a natural fashion the mapping of the fields.

As we have seen, the mapping connecting the model with topological mass generation in four dimensions to the pure topological  $BF$  model is related to similar properties of models in 5-dimensional spacetime which present only the antisymmetric field. In this section we perform one more step in the dimensional reduction program presenting the similar property appearing in the model obtained after the dimensional reduction of the Cremmer-Sherk's action to 3D. The reduced action is given by

$$\begin{aligned} \mathcal{S} &= \mathcal{S}_{top} + \mathcal{S}_{ntop} \\ &= \int d^3x \left( -\frac{m}{6} \varepsilon^{\mu\nu\rho} \varphi H_{\mu\nu\rho} + \frac{m}{2} \varepsilon^{\mu\nu\rho} c_\mu F_{\nu\rho} \right) + \\ &\quad \int d^3x \left( -\frac{1}{4} G^{\mu\nu} G_{\mu\nu} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi \right) \end{aligned}$$

where, after the reduction,

$$\begin{aligned} A_\mu &\rightarrow A_\mu, \varphi, \\ B_{\mu\nu} &\rightarrow B_{\mu\nu}, C_\mu, \\ G_{\mu\nu} &= \partial_\mu C_\nu - \partial_\nu C_\mu. \end{aligned}$$

The mapping to ensure that

$$\mathcal{S}_{top}(\hat{A}_\mu, \hat{B}_{\mu\nu}, \hat{\varphi}, \hat{c}_\mu) = \mathcal{S}_{top}(A_\mu, B_{\mu\nu}, \varphi, c_\mu) + \mathcal{S}_{ntop}(A_\mu, B_{\mu\nu}, \varphi, c_\mu),$$

will be given by

$$\hat{A}_\mu = A_\mu + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} \vartheta_\mu^n, \quad (3.29)$$

$$\hat{B}_{\mu\nu} = B_{\mu\nu} + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} \phi_{\mu\nu}^n, \quad (3.30)$$

$$\hat{\varphi} = \varphi + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} \alpha^n \quad (3.31)$$

$$\hat{c}_\mu = c_\mu + \sum_{n=1}^{\infty} \frac{1}{(2m)^n} k_\mu^n. \quad (3.32)$$

Following the same lines as in the previous section the coefficients are iteratively defined. The first ones are given by

$$\vartheta_\mu^1 = -\frac{1}{2} \varepsilon_{\mu\nu\rho} G^{\nu\rho}, \quad (3.33)$$

$$\vartheta_\mu^2 = (1 - B_1) \partial^\nu F_{\nu\mu},$$

$$\vartheta_\mu^3 = \frac{B_1}{2} \varepsilon_{\mu\alpha\beta} \square G^{\alpha\beta},$$

$$\vartheta_\mu^4 = (-1 - B_1 + B_1^2)(1 - B_2) \square \partial^\nu F_{\nu\mu}, \quad (3.34)$$

$$\phi_{\mu\nu}^1 = -\varepsilon_{\mu\nu\rho} \partial^\rho \varphi,$$

$$\phi_{\mu\nu}^2 = \frac{B_1}{3} \partial^\alpha H_{\alpha\mu\nu},$$

$$\phi_{\mu\nu}^3 = -(1 - B_1) \varepsilon_{\mu\nu\rho} \square \partial^\rho \varphi,$$

$$\phi_{\mu\nu}^4 = (-1 - B_1 + B_1^2) B_2 \square \partial^\alpha H_{\alpha\mu\nu}, \quad (3.35)$$

$$\alpha^1 = -\frac{1}{6} \varepsilon_{\mu\nu\rho} H^{\mu\nu\rho}$$

$$\alpha^2 = (1 - B_1) \square \varphi$$

$$\alpha^3 = \frac{B_1}{6} \varepsilon_{\mu\nu\rho} \square H^{\mu\nu\rho}$$



$$\alpha^4 = (-1 - B_1 + B_1^2)(1 - B_2)\square^2\varphi \quad (3.36)$$

$$\begin{aligned} \beta_\mu^1 &= \frac{1}{2}\varepsilon_{\mu\nu\rho}F^{\nu\rho} \\ \beta_\mu^2 &= B_1\partial^\nu G_{\nu\mu} \\ \beta_\mu^3 &= \frac{1 - B_1}{2}\varepsilon_{\mu\nu\rho}\square F^{\nu\rho} \\ \beta_\mu^4 &= B_2(-1 - B_1 + B_1^2)\square\partial^\alpha G_{\alpha\mu}. \end{aligned} \quad (3.37)$$

It should be clear that the above expression may be summed up leading to expressions that parallel the exact mapping obtained in D=4. Indeed the classical argument of mapping the actions as above depicted allows to obtain the coefficients in dimensionally reduced models from the knowledge of the higher dimensional mapping coefficients. Since the propagators are the inverses of the operators defining the actions their exact mapping in lower dimensions can also be read from the corresponding expressions in higher dimensions. The non-local operators that map the local fields are essentially the same ones.

### 3.1 Cohomological argument

With the aim of giving a cohomological argument to equations (2.2) and (2.6) we use the Batalin-Vilkovisky formalism. Five antifields are required  $(A^{*\mu}, B_{\mu\nu}^*, c^*, \eta^{*\mu}, \rho^*)$ , corresponding respectively to the gauge connection  $A_\mu$ , to the antisymmetric tensorial field  $B_{\mu\nu}$ , the ghost of Faddeev-Popov  $c$ , the ghost which comes from the gauge transformation of  $B_{\mu\nu}$  and to the ghost  $\rho$ . The last ghost appears because of the reducibility of the  $B_{\mu\nu}$  symmetry, which requires, to the complete fixation, one extra ghost.

As mentioned before, the cohomological analysis of the BRST differential operator identifies the terms which can, or cannot, be reabsorbed by field redefinitions [7]. In this context, quantities which do not belong to the cohomology of this operator can be, at the classical level, absorbed by field redefinitions. As we will show now, terms such as  $F_{\mu\nu}F^{\mu\nu}$  and  $H_{\mu\nu\rho}H^{\mu\nu\rho}$ , contained in the action (2.2), are not out of this rule.

To see the triviality of these terms lets write the actions as

$$\Sigma = S + \Sigma_{ant},$$

where

$$\Sigma_{ant} = \int d^4x \left( A_\mu^* \partial^\mu c + B_{\mu\nu}^* \partial^\mu \eta^\nu + \eta_\mu^* \partial^\mu \rho \right).$$

The nilpotent BRST operator is given by

$$\begin{aligned} s = \int d^4x \left( \frac{\delta\Sigma}{\delta A_\mu^*} \frac{\delta}{\delta A^\mu} + \frac{\delta\Sigma}{\delta A^\mu} \frac{\delta}{\delta A_\mu^*} + \frac{\delta\Sigma}{\delta B_{\mu\alpha}^*} \frac{\delta}{\delta B^{\mu\alpha}} + \frac{\delta\Sigma}{\delta B^{\mu\alpha}} \frac{\delta}{\delta B_{\mu\alpha}^*} \right. \\ \left. + \frac{\delta\Sigma}{\delta c^*} \frac{\delta}{\delta c} + \frac{\delta\Sigma}{\delta c} \frac{\delta}{\delta c^*} + \frac{\delta\Sigma}{\delta \eta_\mu^*} \frac{\delta}{\delta \eta^\mu} + \frac{\delta\Sigma}{\delta \eta^\mu} \frac{\delta}{\delta \eta_\mu^*} + \frac{\delta\Sigma}{\delta \rho^*} \frac{\delta}{\delta \rho} + \frac{\delta\Sigma}{\delta \rho} \frac{\delta}{\delta \rho^*} \right). \end{aligned}$$

The action of this operator on the fields and anti-fields is given by:

$$sA_\mu = \partial_\mu c \quad (3.38)$$

$$\begin{aligned}
sB_{\mu\nu} &= \partial_\mu \eta_\nu - \partial_\nu \eta_\mu \\
s\eta_\mu &= \partial_\mu \rho \\
s\mathcal{C} &= s\rho = 0 \\
sA_\mu^* &= \partial^\nu F_{\nu\mu} + \frac{m}{6} \varepsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} \\
sB_{\mu\nu}^* &= -\partial^\rho H_{\rho\mu\nu} + \frac{m}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}, \\
s\eta_\mu^* &= \partial^\nu B_{\mu\nu} \\
s\mathcal{C}^* &= \partial^\mu A_\mu^* \\
s\rho^* &= \partial^\mu \eta_\mu^*
\end{aligned}$$

The triviality of these terms is strongly based in the BRST transformations form of the antifields  $A^{*a\mu}$  and  $B_{\mu\nu}^{*a}$ , given by (3.38). Contracting conveniently the expressions contained in (3.38) with the  $\varepsilon$  - tensorial density we obtain

$$H_{\mu\nu\rho} = \frac{1}{m} (\varepsilon_{\mu\nu\rho\sigma} sA^{*\sigma} - \varepsilon_{\mu\nu\rho\sigma} \partial_\chi F^{\chi\sigma}). \quad (3.39)$$

and

$$F_{\mu\nu} = -\frac{1}{2m} (\varepsilon_{\mu\nu\rho\sigma} sB^{*\rho\sigma} + \varepsilon_{\mu\nu\rho\sigma} \partial_\chi H^{\chi\rho\sigma}) \quad (3.40)$$

The equations (3.39) and (3.40) can be used in a recursive way,

$$H_{\mu\nu\rho} = \frac{1}{m} s \sum_{n=0}^{\infty} \left( \frac{\partial}{m} \right)^{2n} \left( \varepsilon_{\sigma\mu\nu\rho} A^{*\sigma} - \frac{1}{m} H_{\mu\nu\rho}^* \right), \quad (3.41)$$

$$F^{\mu\nu} = -\frac{1}{2m} \varepsilon^{\mu\nu\rho\sigma} s \left\{ B_{\rho\sigma}^* + \frac{1}{m} \sum_{n=0}^{\infty} \left( \frac{\partial}{m} \right)^{2n} \left( \frac{1}{2} \varepsilon_{\alpha\eta\rho\sigma} F^{*\alpha\eta} - \frac{1}{m} \partial^\alpha H_{\alpha\rho\sigma}^* \right) \right\} \quad (3.42)$$

where

$$F_{\mu\nu}^* = \partial_\mu A_\nu^* - \partial_\nu A_\mu^*$$

and

$$H_{\mu\nu\rho}^* = \partial_\mu B_{\nu\rho}^* + \partial_\rho B_{\mu\nu}^* + \partial_\nu B_{\rho\mu}^*.$$

Since  $sH_{\mu\nu\rho} = sF_{\mu\nu} = 0$ , the triviality of the curvatures  $F_{\mu\nu} = s\Lambda_{\mu\nu}$  and  $H_{\mu\nu\rho} = s\Gamma_{\mu\nu\rho}$ , which clearly appears in the equations (3.41) and (3.42), can be used to show that the terms  $\int F^2$  and  $\int H^2$  take the final form

$$\int d^4x F^{\mu\nu} F_{\mu\nu} = s \int d^4x F^{\mu\nu} \Lambda_{\mu\nu}$$

and

$$\int d^4x (H_{\mu\nu\rho} H^{\mu\nu\rho}) = s \int d^4x H^{\mu\nu\rho} \Gamma_{\mu\nu\rho} \quad (3.43)$$

which lead us to conclude that they can be reabsorbed by redefinitions of fields.

The above analysis is similar to that presented in reference [8]. Looking at the BRST transformations of the antifields  $A^*$  and  $B^*$ , given by (3.38), we see that the terms which lead to the

conclusion of the triviality of the curvatures are  $\varepsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}$  and  $\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ . Clearly, these terms are related to the topological term  $\varepsilon BF$ . We see then that, in the cases of Chern-Simons and Cremmer-Sherk, the presence of the topological terms leads to the existence of field iterative redefinitions that we have presented.

## 4 Conclusions

In this work we have studied in some detail a generalization from 2+1D to 3+1D of a procedure that was known to map the MCS field to CS field. In (3+1)D the Cremmer-Sherk model is mapped to the Abelian version of the BF model. This mapping has been established both within an iterative as well as within an exact procedure. One remarkable new aspect that emerges is the presence of a great deal of freedom in the mapping in four dimensions. This freedom has been elucidated as due to the form of the pure topological action which is defined through mixed products of fields. The invariance under rescaling of the fields of the BF type action is responsible for it. Since this kind of action is naturally considered in even dimensional topological models, the non-uniqueness in the mapping should be expected to hold in even dimensions.

The knowledge of the exact mapping provides us with a typical scale, given by the mass parameter  $m$ . The mapping may be used for instance for computing loop variables of the Cremmer-Sherk model using the corresponding expressions of the pure BF model. This suggests to perform the computation in closed fashion without resource to expansions given by the iterative mapping. In any case the mass parameter  $m$  may provide valuable hints to discern in which cases computations using the iterative mapping should or not be considered reliable. It can even provide alternative expansions for instance in direct powers of  $m$  instead of the inverse power series provided by the iterative mapping.

The dimensional reduction arguments here presented relates the mechanism of mapping from more involved actions to structurally minimized models in different dimensions. Besides providing a criterion to fix the mapping, the kinematical dimensional reduction may offer insights as to the low momenta field variables needed in dimensional reductions of high temperature limits in field theory. Actions of the Cremmer-Sherk type are expected to play a role in approaches where current fermions condensates are explicitly controlled with a bosonization scheme. The high temperature limit of QED under this setting will lead to a dimensional reduction paralleling the one here provided.

The cohomological argument here presented should be understood as giving consistency to the mapping among the fields in the iterative approach. Clearly this iterative mapping prevents one from considering the limit  $m \longrightarrow 0$  for which the coefficients would become singular. Nevertheless the explicit knowledge of the exact mapping allows one to consider this limit after summing the series. Indeed this limit in equations (2.21) may be performed smoothly. One should first consider the mass parameter  $m$  of the pure BF model as independent from the corresponding parameter in the Cremmer-Sherk model. The structure constants become simpler non-local functions since they do not present any mass scale in their definitions.

In order to properly appreciate the physical meaning of the mapping, it is important to call attention to the necessity of defining the physical content of a local field theory in terms of the local polynomial algebra of observable fields. The mapping here provided relates two local models each with its physical Hilbert space reconstructed from the Wightmann functions of its own polynomial

algebra[10, 11]. Since the mapping involves non-local functions it should be clear that within the pure BF model there are two Hilbert spaces to be obtained. One Hilbert space is obtained from the local polynomial algebra of fields defined after expressing the Cremmer-Sherk fields non-locally in terms of the pure BF model fields and it should not be confused with the Hilbert space of the pure BF model itself. This later is obtained from its local polynomial algebra of fields. Although constructed with the same model fields the first Hilbert space is not isomorphic to the second one. Instead, it will be isomorphic to the Hilbert space of the Cremmer-Sherk model. The same reasoning goes in the other direction of the mapping. In this context it is clear that neither Hilbert space should be considered as a subspace of the other. It is not a mapping of physical states that is being addressed here but a non-local mapping among the fields.

The generality of the mapping here considered can be further enhanced by introducing arbitrary scalar operators in the definitions of the quadratic non mixed terms of the the vector and antisymmetric fields and considering the parameter  $m$  as an scalar operator acting either on vector or antisymmetric field. This generalized gauge invariant action will be mapped to the pure BF model in a very similar way with the structure functions of the mapping being slightly modified. Furthermore it is to be expected[8] that the introduction of arbitrary gauge invariant interaction terms can be absorbed by considering nonlinear mappings.

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